Strangelets at Finite Temperature

M. G. Mustafa * and A. Ansari *

Institute of Physics, Bhubaneswar-751005, India.

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Abstract

Within the MIT bag model picture of QCD, we have studied the stability of finite size strangelets at finite temperature with baryon number $A \leq 100$. The light quarks u and d are considered massless while mass of the s quark is taken as 150 MeV. Using discrete eigen energies of the non-interacting quarks in the bag, grand canonical partition function is constructed and then the free energy is minimized with respect to the bag radius. In the $T \to 0$ limit clear shell structures are found which persist upto about $T = 10$ MeV. Infact for $A = 6$ the shell structure disappears only at $T > 20$ MeV.

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I. INTRODUCTION

After Witten's conjecture in 1984 [1] that the strange quark matter (SQM) (consisting of roughly equal number of u, d and s quarks) might be absolutely stable compared to iron, several model studies have been made on its stability propertites at zero temperature [2–10] as well as at finite temperatures [11–15]. Particularly, in view of relativistic heavy ion collisions (RHIC) and the possibility of quark-gluon plasma (QGP) formation, it has become interesting to investigate theoretically as well as experimentally the possible existence of a stable or metastable lump of SQM termed as strangelet [4–10]. For example, in Ref. [15] it has been argued that when QGP starts cooling down and hadronization begins u and d quarks combine with anti-strange quarks ($\bar{s}$) producing K^+ and K^0 mesons and in the process leaving enough number of s quarks to condensate along with u and d quarks into small strangelets at finite temperatures. Hopefully the cooling is very fast so that the strangelets can really be formed at sufficiently low temperatures, though finite.

Recently Gilson and Jaffe [10] have considered independent single-particle (sp) shell model approach with quarks confined in a bag with a given bag constant B. The energy of the bag for a given baryon number A is minimised with respect to the bag radius at zero temperature. Even if the bulk energy per baryon $\epsilon_b \geq 950$ MeV, considering u and d massless and mass of s quark $m_s = 150$ MeV, they get shell structures leading to metastability against nucleon decay. Madsen [5,8] has made a shell model versus liquid drop model study for strangelets and gets metastability with life time $\tau \geq 10^{-8}$ second. Boiling and evaporation of SQM at finite temperature has also been studied in the literature [11–13].

Here we essentially want to extend to finite temperature shell model study of strangelets by Gilson and Jaffe [10] with $A \leq 100$. That is we want to study the thermodynamic properties of strangelets (big MIT bag) with a scenario of thermalised hadronic phase after the expansion and cooling of QGP in which the strangelets are embaded at finite temperatures. To take care of the finite size of the strangelets in the best way, we construct a grand canonical partition function using the discrete egien energies of quarks in the bag. As in

most of the calculations in the literature we also consider u and d quarks to be massless and $m_s = 150$ MeV. We do not have any restriction on the strangeness per baryon, *i.e.*, S/A ratio is unrestricted. Besides, strangelets are considered to be charge neutral and flavour eqilibrium is maintained against weak decays [11]. We minimize the free energy for a given A as a function of bag radius R at a given temperature and look for the stable solutions. At present we are not constraining the partition function to be colour-singlet. Since the considered temperatures are rather low, $T \leq 40$ MeV the contributions from gluons and anti-quarks are negligible. Also the temperature dependence of the bag constant B should not be important.

In the next sec. II we present briefly the formalism. In sec. II we shall present our results and discussions along with a brief conclusions in sec. IV.

II. FORMALISM

We construct a small strangelet at finite temperature following Ref. [10] as a gas of non-interacting fermions by filling the bag energy states sequentially, obeying the Pauli exclusion principle. The energy levels for quarks (massless u and d-quarks and massive s-quarks with mass, $m_s = 150$ MeV) are obtained by solving the Dirac equation for a spherical cavity of radius R with linear boundary condition [16]. The obtained sp eigen energies are in the unit of $\hbar c/R$. Unlike in Ref. [10] we consider that there are some electrons in the system to neutralize the electric charge, and the flavour equilibrium is also maintained by weak interactions in which neutrinos are produced. However even if the neutrinos are bound to the system they contribute very little to the energy and pressure of the system. Thus we neglect them entirely. Electrons are also rather scarce but, important for local charge neutrality of the strangelets. Of course, they generate a coloumb barrier but of negligible importance which we will be discussing below.

In a statistical approach like Refs. [17], we construct a grand canonical partition function in terms of participants (quarks, anti-quarks, electrons and positrons) and study the

thermodynamics of finite strangelets. We treat quarks and anti-quarks in the discrete limit, and the electrons and positrons in the continuum limit. In a variational sense, for each A we look for the minimum of the free energy, $F(T, R)$ as function of bag radius R, at various values of temperatures, T. At the minimum of $F(T, R)$ the quark pressure balances the vacuum pressure, B (the bag pressure constant) giving rise to the mass of the strangelet as $4BV$, V being the volume of the system.

Normally in the bag model a zero-point energy is included as a phenomenological term of the form $-Z_0/R$, where fits to hadron spectra [16] indicates the choice $Z_0 = 1.84$ for $B^{1/4} = 145$ MeV. Roughly half of this phenomenological term has a physical basis in centre of mass motion. For reasonable parameter values one finds a significant effect of zero-point energy for $A < 10$, but the term quickly becomes negligible for increasing A, where its contribution to the energy per baryon goes like $A^{-4/3}$. Therefore, because of the great uncertainty, like in Ref. [8], zero-point energy is not included in our calculation. The coloumb energy per baryon is negligible for most strangelets due to the cancellation of u, d and s-quark charges and we ignore it [10]. We also ignore residual perturbative QCD interation following Ref. [2] as for most of the purposes a non-zero value of α_s can be absorbed in the decrease of B provided m_s and bulk binding energy are held fixed. Now, we briefly outline the mathemtical steps involved in our calculation of hot strangelets within the MIT Bag model.

The partition function for quarks and anti-quarks in terms of single-particle energies [17] is defined through

$$\ln \mathcal{Z}_{q(\bar{q})} = \sum_{\alpha} \left\{ \sum_{q=u,d,s} d_{\alpha}^q \ln\left(1 + e^{-\beta(\epsilon_{\alpha}^q - \mu_q)}\right) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\alpha}^{\bar{q}} \ln\left(1 + e^{-\beta(\epsilon_{\alpha}^{\bar{q}} - \mu_{\bar{q}})}\right) \right\}, \qquad (1)$$

$\epsilon_{\alpha}^{q(\bar{q})}$ are single particle energies for quarks(q) and anti-quarks($\bar{q}$), and μ_q are chemical potential for different quark flavours whereas $\mu_{\bar{q}} = -\mu_q$ are those for anti-quarks. $\beta = 1/T$, is inverse of the temperature. $d_{\alpha}^{q(\bar{q})}$ is the degeneracy with $3 \times (2j + 1)$ for each state α designated by angular momentum j and κ, the Dirac quantum number [16], and 3 is the colour factor for each flavour.

As mentioned earlier, we treat electrons(positrons) in the continuum limit and their partition function can be written as

$$\ln \mathcal{Z}_e = \frac{gV}{2\pi^2} \left[\int \epsilon_e^2 \ln \left(1 + e^{-\beta(\epsilon_e - \mu_e)} \right) d\epsilon_e + \int \epsilon_e^2 \ln \left(1 + e^{-\beta(\epsilon_e + \mu_e)} \right) d\epsilon_e \right], \tag{2}$$

where $g = 2$ is the spin degeneracy for $e^-(e^+)$ and μ_e is the chemical potential for e^- and $-\mu_e$ is that for e^+. However, one can neglect the e^+ as it contributes nothing in the temperature range we will be working. Infact, contributions of e^- are also very small.

We shall assume that the weak interaction maintains equilibrium between the different quark flavours through the processes:

$$d \longrightarrow u + e^- + \bar{\nu}_e , \qquad s \longrightarrow u + e^- + \bar{\nu}_e ,$$

$$u + e^- \longleftrightarrow d + \nu_e , \qquad u + e^- \longleftrightarrow s + \nu_e ,$$

$$u + s \longleftrightarrow d + u. \tag{3}$$

However the contributions of electrons to the energy and pressure in the study of equilibrium properties of the strangelets are neglected.

The dynamical chemical equilibrium among the participants yeilds:

$$\mu_d = \mu_s = \mu , \text{and} \; \mu_u = \mu - \mu_e . \tag{4}$$

The local charge neutrality gives a relation among number densities :

$$\frac{2}{3}(n_u - n_{\bar{u}}) - \frac{1}{3}(n_d - n_{\bar{d}}) - \frac{1}{3}(n_s - n_{\bar{s}}) - (n_{e^-} - n_{e^+}) = 0 ,$$

$$2\triangle n_u - \triangle n_d - \triangle n_s - 3\triangle n_e = 0 . \tag{5}$$

The number densities are given as

$$n_i = \frac{N_i}{V} = \frac{T}{V}\frac{\partial}{\partial \mu_i}(\ln \mathcal{Z}_i) , \tag{6}$$

where V is the volume of the system, N_i is the number of each participants, and $i = q, \bar{q}$ and e. The change in number densities are given, respectively, as

$$\triangle n_q = \frac{1}{V} \sum_{\alpha} \left[\sum_q \frac{d_\alpha^q}{\left(e^{\beta(\epsilon_\alpha^q - \mu_q)} + 1\right)} - \sum_{\bar{q}} \frac{d_\alpha^{\bar{q}}}{\left(e^{\beta(\epsilon_\alpha^{\bar{q}} + \mu_q)} + 1\right)} \right], \tag{7}$$

and,

$$\triangle n_e = \frac{g}{2\pi^2} \int d\epsilon_e \left[\frac{1}{\left(e^{\beta(\epsilon_e - \mu_e)} + 1\right)} - \frac{1}{\left(e^{\beta(\epsilon_e + \mu_e)} + 1\right)} \right] \tag{8}$$

A baryon number A is imposed by adjusting quark chemical potentials such that the excess number of q over $\bar{q}$ is $3A$, namely,

$$\triangle N_q = N_q - N_{\bar{q}} = (N_u - N_{\bar{u}}) + (N_d - N_{\bar{d}}) + (N_s - N_{\bar{s}}) = 3A . \tag{9}$$

The energy and free energy of the whole system can be written, respectively, as

$$E(T, R) = T^2 \frac{\partial}{\partial T} (\ln \mathcal{Z}_i) + \mu_q \triangle N_q + BV . \tag{10}$$

and,

$$F(T, R) = -T \ln \mathcal{Z}_i + \mu_q \triangle N_q + BV . \tag{11}$$

where BV is the bag volume energy term [16].

The pressure generated by the participants gas

$$P = - \left(\frac{\partial}{\partial V} F(T, R) \right)_{T, \triangle N_q} , \tag{12}$$

is balanced by the bag pressure constant, B which in turn gives the stability condition of the system. Then, the equilibrium energy of the system as given by (10) is

$$E(T, R) = 4BV , \tag{13}$$

where $T^2 \frac{\partial}{\partial T} (\ln \mathcal{Z}_i) + \mu_q \triangle N_q = 3BV$.

III. RESULT AND DISCUSSION

The physical behaviour of a system at a temperature T is governed by the properties of its free energy. Treating the bag like a many-body system at a temperature T, its stability

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features are studied with the variation of free energy, $F(T, R)$, considering R as a variational parameter. It may be reminded that the single-particle energies of quarks are in the units of $\hbar c/R$ and through this R enters in, for instance, eq.(1). For a fixed value of T, A and μ_u (u-quark chemical potential), we assume a trial R. Then we solve (4) and (5) simultaneously for μ (d and s-quark chemical potential) and μ_e (electron chemical potential). After this we calculate A from number constraint (9) and compare with the chosen value of A. The above procedure is repeated changing μ_u till baryon number constraint (9) is satisfied upto certain accuracy with the chosen value of A. Then for that given T, same A and trial R we calculate $F(T, R)$ of the strangelet using (11). After this with those T and A, the whole procedure is again repeated changing R (keeping it in mind that each time (4) and (5) simultaneously and (9) separately have to be satisfied) until the $F(T, R)$ is minimum obeying the stability condition (12). Now we have a equlibrium value of radius R and energy $E(T, R)$ of a strangelet for a given T and A. Then for a fixed T one can find the stability of strangelets for different values of A.

In Fig. 1 we have plotted the energy per baryon $\epsilon = E/A$ for strangelets as a function of A (upto 100) for $B^{1/4} = 145$ MeV at equilibrium (12), for various $T(0.5, 10, 20, 30, 40$ MeV). The curve for $T = 0.5$ MeV basically represents the zero-temperature results of Gilson and Jaffe [10]. Here, we have chosen $B^{1/4} = 145$ MeV to have the SQM absolutely stable (compared to nucleons) in the bulk phase and we have the bulk energy $\epsilon_b = 850$ MeV. The metastability for strangelets and various decay modes have been studied by Gilson and Jaffe [10] in four different parameter space within the MIT bag model. The strangelets considered here are stable against neutron decay for $A \geq 10$, where $(m_n - \epsilon)$ is the binding energy (relative to neutron mass $m_n \approx 940$ MeV) of the strangelets. With the choice of higher values of B these curves will essentially move up.

Now, one can notice in Fig. 1 that E/A increases with increase of T and until $T = 30$ MeV strangelets ($A \geq 20$) are stable compared to nucleons $i.e.$, $E/A < 940$ MeV. At $T > 35$ MeV, $E/A > 940$ implying there is no stable configurations and the strangelets become unstable compared to nucleons. In the zero temperature limit (due to numerical

problems we take $T = 0.5$ MeV) the E/A approaches a bluk limit at $A \approx 100$, whereas energy grows significantly for low A. For s-quark mass, as expected, shell closures appear for $A = 6, 18, 24, 42, 54, 84, 102, 150, \cdots$ (of course 102 and 150 are not shown in Fig. 1). At every shell closure E/A has a noticeable dip which corresponds to a more stable strangelet compared to the stangelets in the neighbourhood. With higher values of B these shell closures will give rise to metastable strangelets. Also their positions may somewhat change with change of B and m_s. Among these shell closures with low mass strange quark ($m_s = 150$ MeV), $A = 6$ is the most significant shell closure whereas those with higher A are less conspicuous ones. Our results at $T = 0.5$ MeV more or less similar to those of Gilson and Jaffe [10] at $T = 0$. Our calculation differs from that of Gilson and Jaffe [10] as we consider charge neutrality and chemical equilibrium and ignore zero-point energy correction. The most important point is that we are able to see the effect of temperature on shell effects and the latter persists upto about $T = 10$ MeV. Infact, at $A = 6$ the shell effect persists even upto $T = 20$ MeV. Though for higher values of B these states become metastable, the shell features remain unaffected and we have checked it for $B^{1/4} = 160, 175$ MeV.

Now we can analyze how bag states are filled up by quarks. The dynamics is as follows. At $T \to 0$ (of course here $T = 0.5$ MeV) and A upto 3 the fermi surface is such that it only accomodates non-strange $1s_{1/2}$ states and strange $1s_{1/2}$ state (corresponding to massive s-quark) lies outside the fermi surface created by non-strange quarks. At $A = 3$, non-strange $1s_{1/2}$ state saturates with 6 d-quarks, and is half-filled with 3 u-quarks and strange $1s_{1/2}$ state remains empty. This can be seen clearly from Figs. 2 and 3. Upto $A = 3$ there is no strangelets rather they are ud-droplets and their $E/A \geq 940$ MeV (see Fig. 1). In the region $3 < A < 6$, non-strange $1s_{1/2}$ state for d-quarks can not accomodate further as it is already saturated (see Fig. 2). Same feature can also be seen from Fig. 3 where d-fraction is decreasing with increasing A in this region ($3 < A < 6$). The non-strange $1s_{1/2}$ level for u-quarks fills up with constant rate as u-fraction remains constant here and throughout (see Fig. 3). The strange $1s_{1/2}$ level starts accomodating because, now, the strange quark mass is lower than the fermi energy generated by ud-droplets. So, the opening of a new flavour

degree of freedom tends to lower the fermi energy and hence the mass of the strangelets (see Fig. 1). At $A = 6$, $1s_{1/2}$ states for all the three u, d and s-quarks are completely filled and first shell closure occures. This can be seen clearly from Figs. 2 and 3.

After one shell closure, it again becomes favourable to add non-strange quarks to the system. This continues until d-quarks have fully occupied the corresponding non-strange shell. Then again strange levels become energitically favourable to fill them up. When the corresponding strange level saturates, the next shell closure occures. So we find that non-strange and strange states in MIT bag are filled up sequentially obeying Pauli exclusion principle and every shell closure occures due to the completion of a particular strange level (see Figs. 2 and 3).

Now, we discuss how charge neutrality (5) is satisfied in $T \rightarrow 0$ limit (here $T = 0.5$ MeV). Above we have already discussed how levels are filled up in the bag for strangelets. When only non-strange states (u, d states) are filling up (see Figs. 2 and 3) with particular j and κ, corresponding strange state is empty and Fig. 4 indicates that the number of electron is zero. This implies that the charge neutrality (5) is only satisfied among different quark flavours of filled and currently filling states. This situation continues unless d-state is saturated with full occupancy and u-state is just half-filled. Soon after this the half-filled u-state starts filling further and corresponding empty s-state starts accomodating particles (see Figs. 2 and 3). The charge neutrality (5) demands electrons until u and s-states are completely occupied. This feature can be seen from Fig.4. When low lying u, d and s-states are completely filled, a shell closure occures $i.e.$, there is equal number of u, d and s-quarks and charge neutrality implies no electrons. Immediately after a shell closure, feature is same till next shell closure appears because first to fill is next non-strange shell and then strange shell. We can see from Figs. 2, 3 and 4 that this situation recurs atleast upto $A = 100$. The strangelets with closed shells are relatively more stable and less chemically reactive because the decay of an s quark to d quark would be suppressed or forbidden because lowest single-particle levels are fully occupied. We find from Fig. 3 that in the temperature region $20 < T < 30$, the s-fraction is around 0.85 and that of $d \sim 1.15$. However, u-fraction remains

constant throughout at unity at all T. Fig. 4 shows that with the increase of temperature the electron content of the strangelet increases. The increase in the number of electrons indicates higher neutrino emissivity from strangelets. It also improves the heat transport across the bag surface in a system such as a neutron star with the SQM core.

We find that at high temperature ($T > 35$) where strangelets become unstable these are not in flavour equilibrium. So strangelets can decay via weak semileptonic decays, weak radiative decays and electron capture all of which can have $\triangle A = 0$. This is also applicable even at $T \rightarrow 0$ limit. At $T = 0$, E/A determines whether the system is stable against decay into neutron or nuclei. For $E/A < 930$ MeV, strangelets can not decay even into ^{56}Fe nuclei. As long as $E/A < 930$ MeV, emission of an α-particle goes via energy fluctuations but only subject to decay modes which reduce A. For higher temperature, $E/A > 930$ MeV, these fluctuations are not needed and the process can proceed faster. At higher temperature neutron decay might take place if the difference of energy between two strangelets is greater than m_n, $\triangle S = 0(-1)$ and $\triangle A = -1$. Simililarly, for the Λ, Σ, Ξ, Ω decays, one should have $\triangle A = -1$ and $\triangle S = -1$, -1, -2, -3, respectively.

IV. CONCLUSION

Considering non-interacting massless u and d-quarks and massive s-quarks (m_s=150 MeV) confined in an MIT bag at finite temperature thermodynamic properties of finite size strangelets with baryon number $A \leq 100$ have been studied following the method of quantum statistics. As discussed in the text, the grand canonical partition function at $T = 0.5$ MeV describes rather well the $T \rightarrow 0$ limit properties of the strangelets showing shell structures and the possibility of stable or metastable strangelets depending on the employed value of the bag constant B. As far we are aware, this is seen for the first time that the shell structure gets washed away (no possibility of metastable states) only at $T \approx 10$ MeV. Infact for $A = 6$ the shell structure vanishes only at $T \geq 20$ MeV. We find that for $B^{1/4} = 145$ MeV which is a physically accepted value, the strangelets remain stable against nucleon decay till $T = 30$

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MeV.

Next we are planning to repeat this calculation with the use of a colour-singlet partition function which would incorporate the most important aspect of QCD interaction.

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*   e-mail: mustafa / ansari@iopb.ernet.in

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FIGURES

FIG. 1.   Energy per baryon (E/A) as a function of baryon number A for different temperatures (T =0.5, 10, 20, 30, 40 MeV) with $B^{1/4} = 145$ MeV and $s-$quark mass m_s=150 MeV.

FIG. 2. Number of quarks (d and $s-$quarks) as a function of baryon number A for different temperatures (T =0.5, 10 MeV) with $B^{1/4} = 145$ MeV and $s-$quark mass m_s=150 MeV.

FIG. 3. Quark-fraction as a function of baryon number A for different temperatures (T =0.5, 10, 20, 30 MeV) with $B^{1/4} = 145$ MeV and $s-$quark mass m_s=150 MeV.

FIG. 4.   Number of electrons as a function of baryon number A for different temperatures (T =0.5, 10, 20, 30 MeV) with $B^{1/4} = 145$ MeV and $s-$quark mass m_s=150 MeV.